

# Chirally symmetric but confining dense and cold matter.

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The folklore tradition about the QCD phase diagram is that at the chiral restoration phase transition at finite density hadrons are deconfined and there appears the quark matter. We address this question within the only known exactly solvable confining and chirally symmetric model. It is postulated within this model that there exists linear Coulomb-like confining interaction. The chiral symmetry breaking and the quark Green function are obtained from the Schwinger-Dyson (gap) equation while the color-singlet meson spectrum results from the Bethe-Salpeter equation. We solve this model at  $T=0$  and finite chemical potential  $\mu$  and obtain a clear chiral restoration phase transition at the critical value  $\mu_{cr}$ . Below this value the spectrum is similar to the previously obtained one at  $\mu = 0$ . At  $\mu > \mu_{cr}$  the quarks are still confined and the physical spectrum consists of bound states which are arranged into a complete set of exact chiral multiplets. This explicitly demonstrates that a chirally symmetric matter consisting of confined but chirally symmetric hadrons at finite chemical potential is also possible in QCD. If so, there must be nontrivial implications for astrophysics.

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## INTRODUCTION

The structure of the QCD phase diagram is one of the keys for our understanding of QCD, strongly interacting matter and has direct astrophysical implications. It is a subject of a significant effort within the heavy ion collision programs at different labs. We know that in the vacuum as well as at low temperature and density QCD is realised in the confining hadronic phase with broken chiral symmetry. At high temperature and low density the system is deconfined and chiral symmetry is restored. Above the critical temperature one experimentally observes a strongly interacting plasma with dynamics that is not yet well understood. Only at the very high temperatures one possibly approaches a perturbative regime of the weakly interacting quark-gluon plasma. Just in the opposite limit of the asymptotically large densities and small temperature one believes that the perturbative gluonic interaction is also adequate and consequently one has, depending on the number of active flavors the color superconducting phase (the color-flavor locking phase with  $N_f = N_c$ ).

There is a folklore tradition that upon increasing temperature and density from zero the deconfining and chiral restoration transitions coincide and beyond the semicircle in the  $T - \mu$  plane one obtains a deconfining and chirally restored matter. From the theoretical side we know from the 't Hooft anomaly matching conditions [1] that at zero temperature and density in the confining phase chiral symmetry must be necessarily broken in the vacuum. The converse is not required, of course. Whether the chiral restoration and deconfinement transitions coincide at finite temperature and zero chemical potential can be answered on the lattice. It was believed that the same temperature triggers both transitions [2],

though recently there have appeared indications that it could be not so [3]. Unfortunately lattice cannot help us at finite chemical potential due to the notorious sign problem. Hence one has to rely on models of QCD. Typically models that give us information about the phase structure are of the Nambu and Jona-Lasinio type, for an overview and references see [4]. An obvious drawback of all these models is that they are not confining. Then there is no basis to conclude that above the chiral restoration point at finite chemical potential one obtains a chirally symmetric and deconfining quark matter.

Recently McLerran and Pisarski presented qualitative large  $N_c$  arguments showing that at a reasonably large chemical potential and low temperature there might exist a confining but chirally symmetric phase [5]. This suggestion would dramatically change the existing paradigm and requires urgent efforts to verify whether it could be indeed the case. This suggestion is in conflict with the naive intuition that once the hadrons are confined chiral symmetry should be necessarily broken. There are no such limitations from the 't Hooft anomaly matching conditions at finite density, however, and such an intuition is in fact an artefact of existing simple models of chiral symmetry breaking and confinement [6, 7].

Clearly we cannot solve QCD and answer this question from first principles. What can be done is to verify this conjecture within a solvable manifestly confining and chirally symmetric model. Such a model could provide insight into physics that would drive such an unexpected situation. If confirmed within such a model, this scenario could also be realised in QCD and further theoretical and experimental efforts to clarify this important question would be required.

There exists only one known manifestly chirally-symmetric and confining model in four dimensions that

is solvable [8], sometimes called Generalized Nambu and Jona-Lasinio model (GNJL). This model can be considered as a generalization of the 1+1 dimensional 't Hooft model, that is QCD in the large  $N_c$  limit [9]. Once the gauge is properly chosen in 1+1 dimensions the Coulomb interaction becomes a linear confining potential. Then this potential properly represents gluonic degrees of freedom in 1+1 dimensions. In four dimensions there are different gluonic interactions and it is hopeless to solve even large  $N_c$  QCD with full gluodynamics. It is postulated within the GNJL model that there exists a linear confining potential of the Coulomb type in four dimensions [25]. This model represents a simplification of large  $N_c$  QCD. The model is exactly solvable. The chiral symmetry breaking and the properties of the Goldstone bosons can be obtained from the solution of the Schwinger-Dyson and Bethe-Salpeter equations [10, 11, 12, 13, 14, 15]. The complete spectrum of  $\bar{q}q$  mesons has been obtained in refs. [16, 17], which exhibits restoration of chiral symmetry in excited hadrons, for a review see [18].

It is instructive to review the chiral symmetry and confining properties of quarks in this model, which are similar to those in the 't Hooft model. Upon solving the Schwinger-Dyson equation for the quark Green functions one obtains chiral symmetry breaking in the vacuum, i.e. a nonzero quark condensate and a dynamical mass  $M(q)$  for the quarks, that is strongly momentum dependent and vanishes at large momenta. This single quark Green function is infrared-divergent, which means that the single quark cannot be observed and hence is confined. The infrared divergence is canceled exactly in the Bethe-Salpeter equation for the observable color-singlet hadrons and the physical spectrum consists only of color-singlet hadrons. This model allows for a solution at finite chemical potential and therefore can be used as a tool to check whether it is possible or not to have confining but chirally symmetric matter at finite chemical potential. Below we solve it and obtain a complete spectrum of mesons at zero temperature and finite quark chemical potential. We demonstrate that below the critical chemical potential,  $\mu_{cr}$ , the spectrum is similar to the one previously obtained at zero chemical potential [16, 17]. At  $\mu = \mu_{cr}$  the chiral restoration phase transition happens. At  $\mu > \mu_{cr}$  hadrons are still confined and the spectrum represents a complete set of chiral multiplets. This explicitly demonstrates that a chirally symmetric but confining phase exists within this model and opens a perspective to study such a possibility in QCD.

### CHIRAL SYMMETRY BREAKING AND BETHE-SALPETER EQUATION FOR MESONS IN VACUUM

Throughout this paper we work in the chiral limit. The two flavor version of the model is considered. The global

chiral symmetry of the model is  $U(2)_L \times U(2)_R$ , because in the large  $N_c$  world the axial anomaly is absent. The model is described in great detail in references [17, 18] so here we give only an overview of this model in vacuum. It is postulated within the model that there is only a linear instantaneous Lorentz-vector interquark potential that has a Lorentz structure of the Coulomb potential, i.e., it is a density-density interaction:

$$K_{\mu\nu}^{ab}(\vec{x} - \vec{y}) = g_{\mu 0} g_{\nu 0} \delta^{ab} V(|\vec{x} - \vec{y}|); \quad \frac{\lambda^a \lambda^a}{4} V(r) = \sigma r, \quad (1)$$

where  $a, b$  are color indices. The Fourier transform of this potential and any loop integral are infrared-divergent. Hence infrared regularization is required and any physical observable, such as a hadron mass, must be independent of the infrared regulator  $\mu_{IR}$  in the infrared limit (i.e. when this regulator approaches 0,  $\mu_{IR} \rightarrow 0$ ). There are several physically equivalent ways to perform this infrared regularization. Here we follow Ref. [11] and define the potential in momentum space as

$$V(p) = \frac{8\pi\sigma}{(p^2 + \mu_{IR}^2)^2}. \quad (2)$$

Then this potential in the configuration space contains the required  $\sigma r$  term, the infrared-divergent term  $-\sigma/\mu_{IR}$  as well as terms that vanish in the infrared limit.

The Dirac operator for the dressed quark is

$$D(p_0, \vec{p}) = iS^{-1}(p_0, \vec{p}) = D_0(p_0, \vec{p}) - \Sigma(p_0, \vec{p}), \quad (3)$$

where  $D_0$  is the bare Dirac operator. Parametrising the self-energy operator in the form

$$\Sigma(\vec{p}) = A_p + (\vec{\gamma} \cdot \hat{\vec{p}})[B_p - p], \quad (4)$$

where functions  $A_p$  and  $B_p$  are yet to be found, the Schwinger-Dyson equation for the self-energy operator in the rainbow approximation, which is valid in the large  $N_c$  limit for the instantaneous interaction, is reduced to the nonlinear gap equation for the chiral angle  $\varphi_p$ ,

$$A_p \cos \varphi_p - B_p \sin \varphi_p = 0, \quad (5)$$

where

$$A_p = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} V(\vec{p} - \vec{k}) \sin \varphi_k, \quad (6)$$

$$B_p = p + \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} (\hat{\vec{p}} \cdot \hat{\vec{k}}) V(\vec{p} - \vec{k}) \cos \varphi_k. \quad (7)$$

The functions  $A_p, B_p$ , i.e. the quark self-energy, are divergent in the infrared limit,

$$A_p = \frac{\sigma}{2\mu_{\text{IR}}} \sin \varphi_p + A_p^f, \quad (8)$$

$$B_p = \frac{\sigma}{2\mu_{\text{IR}}} \cos \varphi_p + B_p^f, \quad (9)$$

where  $A_p^f$  and  $B_p^f$  are infrared-finite functions. This implies that the single quark cannot be observed and the system is confined. However, the infrared divergence cancels exactly in the gap equation (5) so this equation can be solved directly in the infrared limit [26]. The chiral symmetry breaking is signalled by the nonzero quark condensate and by the dynamical momentum-dependent mass of quarks

$$\langle \bar{q}q \rangle = -\frac{N_C}{\pi^2} \int_0^\infty dp p^2 \sin \varphi_p, \quad M(p) = p \tan \varphi_p. \quad (10)$$

The dynamical mass is finite at small momenta and vanishes at large momenta. Both these quantities were first obtained in ref. [10] and repeatedly reconfirmed in all subsequent works on this model [11, 12, 13, 14, 15, 16, 17]. The numerical value of the quark condensate is  $\langle \bar{q}q \rangle = (-0.231\sqrt{\sigma})^3$ .

The homogeneous Bethe-Salpeter equation for a quark-antiquark bound state in the rest frame with the instantaneous interaction is

$$\begin{aligned} \chi(m, \vec{p}) &= -i \int \frac{d^4 q}{(2\pi)^4} V(|\vec{p} - \vec{q}|) \gamma_0 S(q_0 + m/2, \vec{p} - \vec{q}) \\ &\times \chi(m, \vec{q}) S(q_0 - m/2, \vec{p} - \vec{q}) \gamma_0. \end{aligned} \quad (11)$$

Here  $m$  is the meson mass and  $\vec{p}$  is the relative momentum. The Bethe-Salpeter equation can be solved by means of expansion of the vertex function  $\chi(m, \vec{p})$  into a set of all possible independent amplitudes consistent with  $I, J^{PC}$ . Then the Bethe-Salpeter equation transforms into a system of coupled equations [17]. The infrared divergence cancels exactly in these equations and they can be solved either in the infrared limit or for very small values of the infrared regulator.

In the limit of vanishing dynamical quark mass  $M(p) = 0$  (i.e.,  $\varphi_p = 0$ ), the Bethe-Salpeter equation transforms into systems of coupled equations that exactly fall into a complete set of chiral multiplets [17]:

$$\mathbf{J} = \mathbf{0}$$

$$\begin{aligned} (1/2, 1/2)_a &: 1, 0^{-+} \longleftrightarrow 0, 0^{++} \\ (1/2, 1/2)_b &: 1, 0^{++} \longleftrightarrow 0, 0^{-+}, \end{aligned} \quad (12)$$

$$\mathbf{J} = 2\mathbf{k}, \quad \mathbf{k}=1, 2, \dots$$

$$\begin{aligned} (0, 0) &: 0, J^{--} \longleftrightarrow 0, J^{++} \\ (1/2, 1/2)_a &: 1, J^{-+} \longleftrightarrow 0, J^{++} \\ (1/2, 1/2)_b &: 1, J^{++} \longleftrightarrow 0, J^{-+} \\ (0, 1) \oplus (1, 0) &: 1, J^{++} \longleftrightarrow 1, J^{--} \end{aligned} \quad (13)$$

$$\mathbf{J} = 2\mathbf{k}-1, \quad \mathbf{k}=1, 2, \dots$$

$$\begin{aligned} (0, 0) &: 0, J^{++} \longleftrightarrow 0, J^{--} \\ (1/2, 1/2)_a &: 1, J^{+-} \longleftrightarrow 0, J^{--} \\ (1/2, 1/2)_b &: 1, J^{--} \longleftrightarrow 0, J^{+-} \\ (0, 1) \oplus (1, 0) &: 1, J^{--} \longleftrightarrow 1, J^{++} \end{aligned} \quad (14)$$

Here  $I, J^{PC}$  is a standard notation for isospin  $I$ , spin  $J$  as well as for both spatial and charge parities  $P$  and  $C$ . The sign  $\longleftrightarrow$  indicates that both given states belong to the same representation and that the Bethe-Salpeter equations for these states are *identical*.

The complete spectrum of the  $\bar{q}q$  mesons in the vacuum can be found in refs. [16, 17] and exhibits a fast chiral restoration with increasing  $J$ . A reason for this restoration is that with larger  $J$  the radial wave function vanishes at small relative momenta, and hence the large chiral symmetry breaking dynamical mass  $M(p)$  becomes irrelevant and asymptotically at  $J \rightarrow \infty$  the Bethe-Salpeter equation exactly decouples into a set of equations (13)-(14).

## INCLUSION OF A FINITE QUARK CHEMICAL POTENTIAL AT ZERO TEMPERATURE

### The gap equation

Now we are in a position to include into this model a finite quark chemical potential at zero temperature. We can straightforwardly do it only at zero temperature because to employ thermodynamics at finite temperature one would need to know a temperature dependence of the confining interaction. In addition there is no screening of the confining interaction at zero temperature in the large  $N_c$  limit since in this limit there are no vacuum quark loops and no the particle-hole modifications of the gluonic propagator due to Debye screening [5].

The effect of a finite quark chemical potential at zero temperature reduces to the Pauli blocking of all occupied levels. Denoting the Fermi momentum of quarks as  $p_f$  one has to replace the vacuum density matrix  $v(\vec{p})v^\dagger(\vec{p})$  by the density matrix in the medium with all quark levels occupied up to the Fermi momentum:

$$\rho(\vec{p}) = \Theta(p_f - p) u(\vec{p}) u^\dagger(\vec{p}) + v(\vec{p}) v^\dagger(\vec{p}). \quad (15)$$

Hence one has to remove from the integration both in the Schwinger-Dyson (gap) and Bethe-Salpeter equations all

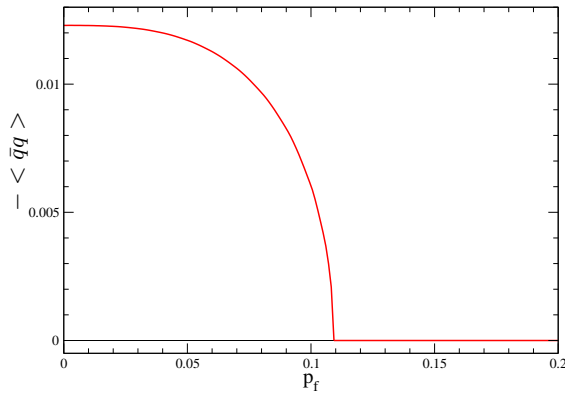


FIG. 1: Quark condensate in units of  $\sigma^{3/2}$  as a function of the Fermi momentum, which is units of  $\sqrt{\sigma}$ .

quark momenta below  $p_f$  since they are Pauli-blocked. The modified gap equation at any  $p > p_f$  is then the same as in (5) - (7), but the integration starts not from  $k = 0$ , but from  $k = p_f$ . The points  $p < p_f$  are irrelevant for the hadron which is always on top of the Fermi sea. Similar, in the expression (10) for the quark condensate the lower limit of the integration is now  $p_f$ , rather than 0. As in the vacuum, we are in the position to solve numerically the nonlinear gap equation at arbitrary  $p_f$ , study the chiral restoration phase transition at the critical  $p_f^{cr}$ , and, given a dressed quark Green function, solve the Bethe-Salpeter equation for the spectrum below and above the critical Fermi momentum.

In Fig. 1 we show the quark condensate  $\langle \bar{q}q \rangle$  as a function of  $p_f$ . One observes an obvious phase transition at the critical Fermi momentum  $p_f^{cr} = 0.109\sqrt{\sigma}$ . This is similar to what has been observed within the 't Hooft model [19]. The reason for this chiral symmetry restoration phase transition is the same as in the standard Nambu and Jona-Lasinio model [20] or in the 't Hooft model [19]: Once a considerable part of the quark self-energy interaction is removed by the Pauli blocking, then there is not enough strength in the gap equation to generate a nontrivial solution with the broken chiral symmetry in the vacuum.

In order to see this physics explicitly consider solutions of the gap equation for the chiral angle  $\varphi_p$  at different fixed Fermi momenta  $p_f$ , that are shown in Fig. 2. One obviously observes a universal approaching of the nontrivial solution of the gap equation  $\varphi_p \neq 0$  to its trivial value  $\varphi_p = 0$  once the Fermi momentum  $p_f$  approaches a critical value  $p_f^{cr}$ . Note that the infrared point in the gap equation (5)-(7) is  $\vec{k} = \vec{p}$  (i.e., where the argument of the potential approaches 0). It directly follows from the self-energy integrals (6)-(7). For any  $p > p_f$  this point is *always* within the integration interval from  $k = p_f$  to  $k = \infty$ . However, while the linear confining potential is infrared-singular itself, the gap equation (5) is not, be-

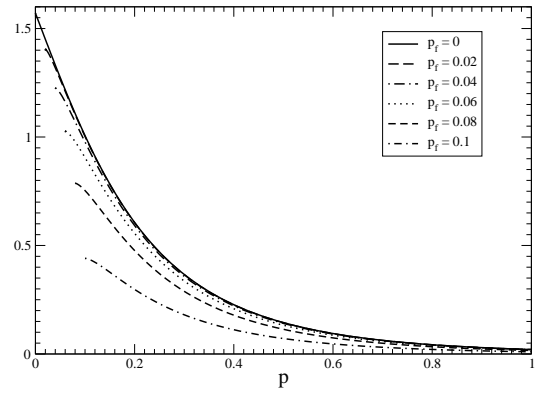


FIG. 2: Chiral angle  $\varphi_p$  as a function of the momentum  $p$  at different fixed values of the Fermi momentum  $p_f$  below the critical value. Both momenta are units of  $\sqrt{\sigma}$ .

cause the infrared divergence cancels out exactly, as it follows from (8)-(9). Hence the contribution to the gap equation comes not only from the vicinity of the infrared point  $\vec{p} = \vec{k}$ , but actually from the whole integration interval. Once a critical part of this integration interval is removed by the Pauli blocking, the nontrivial chiral symmetry breaking solution of the gap equation abruptly disappears. A  $p_f$  dependence of the dynamical mass  $M(p)$  is shown in Fig. 3.

Summarizing, we stress that the symmetry gets restored in our case not because the potential gets screened. In the large  $N_c$  limit it cannot be screened and the string tension  $\sigma$  remains constant at any  $p_f$ . It gets restored exclusively due to the Pauli blocking that reduces a "strength" of the gap equation below the critical value. At each fixed value of  $\sigma$  there always exists such a critical reduction of the integration interval due to the Pauli blocking so that the nontrivial chiral symmetry breaking solution vanishes. This directly follows from the fact that *all* dimensional quantities in our task are given in units of  $\sigma$  and all results are valid for any  $\sigma$ , large or small.

Above the critical value  $p_f^{cr}$  there is no nontrivial chiral symmetry breaking solution and the only solution is trivial,  $\varphi_p = 0$ . This follows from the numerical integration of the highly nonlinear gap equation. It would be difficult, if impossible, to obtain this critical Fermi momentum analytically.

Hence the chiral symmetry gets restored,  $\varphi_p = 0$ , with vanishing quark condensate,  $\langle \bar{q}q \rangle = 0$ , and dynamical mass of quarks,  $M(q) = 0$ , as it follows from (10). At  $\varphi_k = 0$  the Lorentz-scalar self-energy of quarks vanishes identically,  $A_p = 0$ , see eq. (6).

What crucially distinguishes this model from the NJL model is that the quark is still confined, even in the chirally restored phase. This is because for any  $p > p_f$  the Lorentz spatial-vector self-energy integral  $B_p$  does not vanish at  $\varphi_k = 0$  (see eq. (7)) and is in fact infrared-divergent, see (9). Hence the single quark is removed

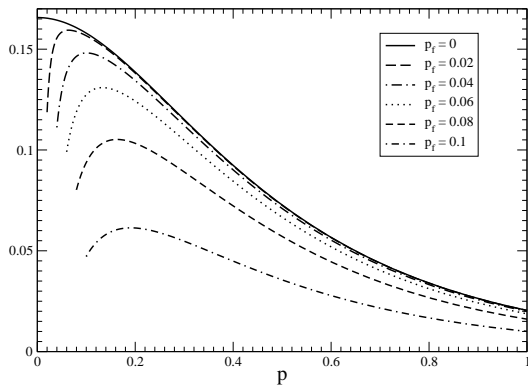


FIG. 3: Dynamical mass of quarks  $M(p)$  as a function of the momentum  $p$  at different fixed values of the Fermi momentum  $p_f$  below the critical value. Both momenta are units of  $\sqrt{\sigma}$ .

from the spectrum at any chemical potential.

### Meson spectrum

This infrared divergence of the single quark Green function cancels exactly, however, in the color-singlet quark-antiquark system [17] and the bound state mesons are finite and well defined quantities. Like in the gap equation, the only modification of the Bethe-Salpeter equation is that the integration in  $q$  starts not from 0, but from  $q = p_f$ .

The complete spectrum of  $\bar{q}q$  mesons at the Fermi momentum  $p_f$  essentially below the critical value,  $p_f = 0.05\sqrt{\sigma}$ , is shown on Fig. 4. This spectrum is similar to the previously obtained one in the vacuum in refs. [16, 17]. Four Goldstone bosons with the quantum numbers  $I = 1, 0^{-+}$  and  $I = 0, 0^{-+}$  are well seen. We remind that there are no vacuum fermion loops in this large  $N_c$  model and hence the  $U(1)_A$  symmetry is broken only spontaneously. The spectrum exhibits approximate restoration of the chiral symmetry in excited hadrons, for details we refer to [17] and for a review to ref. [18].

The spectrum above the critical Fermi momentum, i.e. for  $p_f = 0.2\sqrt{\sigma}$ , is shown in Fig. 5. This spectrum is qualitatively different as compared to Fig. 4. All the states are in *exact* chiral multiplets. A fundamental symmetry reason of this degeneracy is rather obvious: When the vacuum is chiral-invariant, i.e., we are in the Wigner-Weyl mode, the bound state spectrum (if it exists) can only be in exact chiral multiplets [21]. In our case in the Wigner-Weyl mode the quarks are confined and the bound states do exist. Consequently the resulting color-singlet spectrum is manifestly chirally-symmetric. This also directly follows from the analytical symmetry properties of the Bethe-Salpeter equation in the chiral symmetry limit  $M(p) = 0$  [17].

Though the chiral symmetry is manifestly restored,

one observes *finite-energy well defined hadrons*. Obviously the mass generation mechanism in these hadrons has nothing to do with the chiral symmetry breaking and is not related with the quark condensate. The mass generation mechanism for these chirally symmetric hadrons comes from the manifestly chirally symmetric dynamics and is similar to the high-lying states in the chiral symmetry broken phase on Fig. 4.

On Fig. 6 we show an evolution of the ground state pion and the sigma mesons as a function of the Fermi momentum.

### CONCLUSIONS

As a conclusion we have demonstrated that it is possible to have at finite chemical potential a confining but chirally symmetric matter consisting of chirally symmetric hadrons. For that we used the only known exactly solvable model that is confining and manifestly chirally symmetric. Certainly this model is not QCD and perhaps confinement mechanism in QCD is more complicated. However, what we have shown, is "a matter of principle demonstration". We have also clarified perhaps a generic mechanism of the phenomenon. Namely, the chiral symmetry restoration means that the Lorentz-scalar part of the quark self-energy vanishes. But it does not require yet that the other parts of the quark self-energy vanish either. If they do not and the quark is still off-shell, then it cannot be observed and such a single quark is confined. At the same time the color-singlet hadrons are well-defined and finite on-shell systems.

Whether this happens in QCD or not is still an open question but it definitely suggests that there are no reasons to believe that deconfinement and chiral restoration necessarily coincide at finite density. If such a chirally symmetric but confining phase does exist in QCD, then it will imply dramatic modifications of the QCD phase diagram. Schematically this diagram would look like on Fig. 7.

It will also have significant implications for astrophysics: The interactions of these chirally-symmetric hadrons can be only of short-range as they decouple from the Goldstone bosons and their weak decay rate is quite different since their axial charge vanishes [22].

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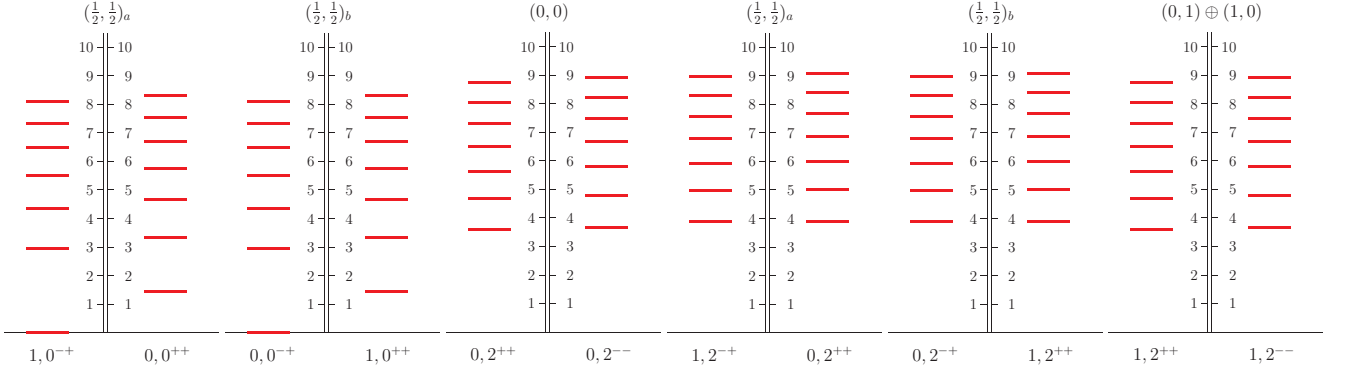


FIG. 4: Spectra for  $p_f = 0.05\sqrt{\sigma}$ . For  $J = 0$  only the two  $(\frac{1}{2}, \frac{1}{2})$  multiplets are present (left two panels). For  $J > 0$  there are also the  $(0, 0)$  and  $(0, 1) \oplus (1, 0)$  multiplets. In the remaining four panels we show all multiplets for  $J = 2$ . Masses are in units of  $\sqrt{\sigma}$ . Meson quantum numbers are  $I, J^{PC}$ .

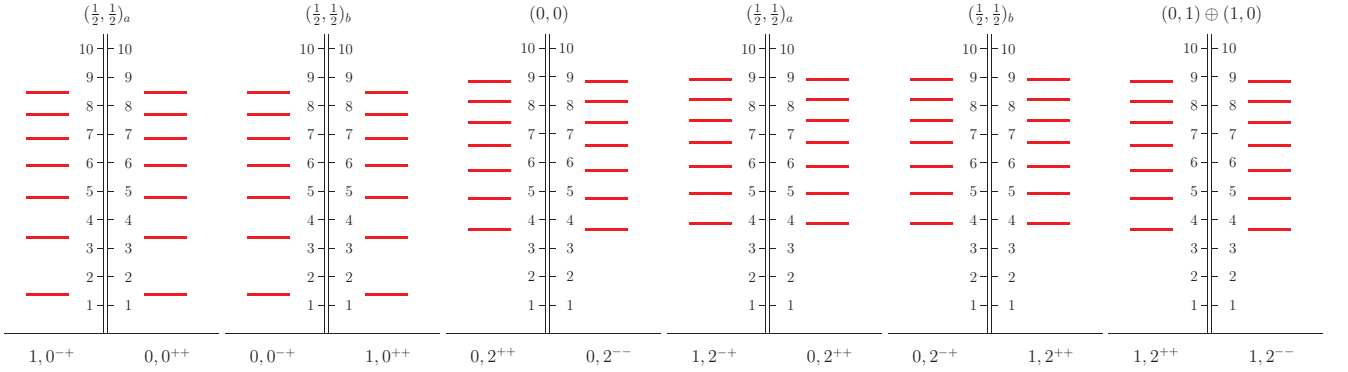


FIG. 5: As Fig. 4 but for Fermi momentum  $p_f = 0.2\sqrt{\sigma}$ .

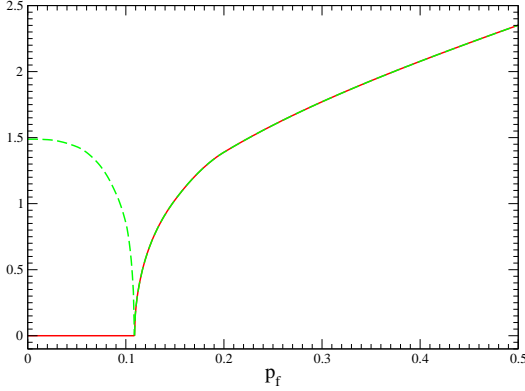


FIG. 6: Masses of the pseudoscalar (solid) and scalar (dashed) mesons in units of  $\sqrt{\sigma}$  as functions of the Fermi momentum in the same units.

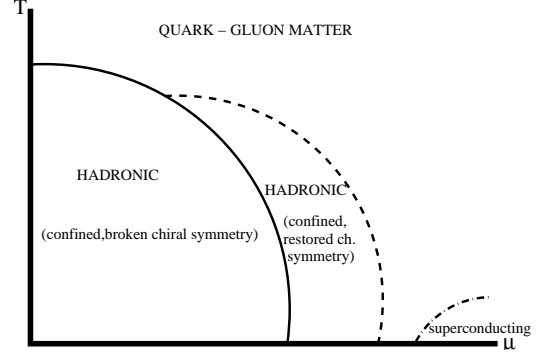


FIG. 7: Schematic possible phase diagram in the  $T - \mu$  plane.

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- [25] Actually there is a number of works that derive such a linear rising confining potential of the Coulomb type in the Coulomb gauge QCD [23] and that connect this potential with the Gribov-Zwanziger scenario for confinement in the Coulomb gauge [24].
- [26] Actually we have solved the gap equation with small values of  $\mu_{\text{IR}} > 0$  in the potential for the IR-divergent functions  $A_p$  and  $B_p$  by iteration on a mesh of a finite number of points in momentum space. In both integrals the angular integrations can be performed analytically. For the remaining numerical calculations of the radial integrals special care has been taken with respect to the vicinity of  $k = p$  where the integrands have sharp peaks. The results for all IR-finite quantities have then been extrapolated numerically to  $\mu_{\text{IR}} = 0$ .